Astronomy 45. Introduction to Astrophysics

Study and Review Problems

These problems need not be turned in, and I definitely don't expect anyone to do them all! They are distributed as an aid for study or review. My recommendation would be to read each problem and make sure you know the basic concepts and equations that would (in principle) let you solve it.

If you do work some of the problems to completion, and want to turn in your solutions for "extra credit," feel free to do this.

Problems marked "Abell" are from the book "Abell's Exploration of the Universe," Seventh Edition, by Morrison, Wolff, and Fraknoi.

I. Geometry and Kinematics

1. For a telescope with 1 arcsec resolution, what is the size of the smallest resolvable feature on Mars (a) when it is closest to the Earth, and (b) when it is farthest from the Earth?

2. Draw the appearance of the moon (phase, orientation, etc.) relative to the horizon just after it rises at 3:00 p.m. on a particular day. Do this (a) for an observer on the equator, and (b) for an observer at 45° N latitude.

3. The wavelength of a particular spectral line is normally 656.3 nm; in a certain star, the line appears at 656.5 nm. How fast is that star moving in our line of sight? Is its motion one of approach or recession? (Abell)

4. Suppose a star at a distance of 10 pc has a radial velocity of 150 km/s. By what percentage does its distance change in 100 years? (Abell)

5. A star has a proper motion of 2.5 arcsec per year, a parallax of 0.32 arcsec, and a radial velocity of 60 km/s away from us. (a) What is the magnitude of its 3-velocity with respect to the Earth's rest frame? (b) What is the angle between the direction of its velocity and the line of sight from Earth?

II. Magnitude Scale

6. As seen from the Earth, the Sun has an apparent magnitude of about -26. What is the apparent magnitude of the Sun as seen from Saturn, about 10 AU distant? (Abell)

7. Find the absolute magnitude of each of the following stars of apparent magnitude m and distance r:

   a. $m = 7; r = 10\,\mathrm{pc}$
   b. $m = 20; r = 100\,\mathrm{pc}$
   c. $m = 0; r = 100\,\mathrm{pc}$
   d. $m = 3.5; r = 5\,\mathrm{pc}$
   e. $m = 17.5; r = 20\,\mathrm{pc}$
   f. $m = -5; r = 1/10\,\mathrm{pc}$ (there is no such star). (Abell)

8. What are the distances of stars of the following apparent magnitudes m and absolute magnitudes M?

   a. $m = 10; M = 5$
   b. $m = 5; M = 10$
   c. $m = 13.5; M = 15$
   d. $m = 20; M = 10$
   e. $m = -26.5; M = 5$ (Abell)

III. Instrumentation

9. If the Moon can be photographed in 1 s with a telescope of 50-cm aperture and 100-cm focal length, how long a time would be required to photograph the Moon with a telescope of 100-cm aperture and 400-cm focal length? (Abell)

10. Suppose that all stars emitted exactly the same total amount of light and that stars were distributed uniformly throughout space. Show that the depth in space to which stars could be observed would (ideally) be proportional to the aperture of the telescope used. (Abell)

11. What is the magnification of an f/8 telescope of $6''$ aperture with an eyepiece of 3 mm focal length? (I actually own such a telescope.)

IV. Radiation

12. Two stars are at the same distance. The two stars have identical diameters. One has a temperature of 5800 K; the other has a temperature of 2900 K. Which is brighter? How much brighter is it in Bolometric Magnitudes? (Abell)

13. Is there a magnitude band (U, B, V, R, I, etc.) in which the *other* star in problem 12 would be brighter? If so, which?

14. If the emitted infrared radiation from Pluto has a wavelength of maximum intensity at 50,000 nm, what is the temperature of Pluto? (Abell)

15. What is the temperature of a star with a wavelength of maximum light of 290 nm? (Abell)

16. Consider a giant star with a radius of 1 AU and a temperature of 3000 K. How much more energy does this star emit than an ordinary star with the same temperature but a radius of only 150,000 km (0.001 AU)? What is the difference in the absolute magnitudes of the two stars? (Abell)

17. Assume the wavelength of maximum light of the Sun to be exactly 500 nm, its temperature exactly 6000 K, and its absolute bolometric magnitude exactly 5.0. Another star has its wavelength of maximum light at 10,000 nm. Its apparent visual magnitude is 15.5, its bolometric correction (that is, $M_V - M_{\rm bol}$) is 0.5, and its parallax is 0.01 arcsec. What is its radius in terms of the Sun's?

    Answer: $R/R_\odot = 0.4$. (Abell)

18. What is the force (in dynes) on a mirror due to a 1 watt laser beam reflecting at normal incidence? Does your answer depend on the wavelength λ of the laser? Why or why not?

19. Assume that the solar interior is all at a temperature of $T = 10^7$ K, and assume that a typical photon has an energy of kT. (These assumptions are not too far off.) Are there more photons, or more particles, in the Sun? By what factor?

20. If your eye is capable of seeing a star that delivers 10^3 visual band photons per second to your pupil (8 mm diameter), what is the implied faintest visible magnitude m_V?

21. How many watts are emitted by one square meter of solar surface? Do this two ways: (1) Starting with the total solar luminosity, and (2) starting with the Sun's effective temperature.

22. How long does it take to get a suntan from an ordinary light bulb? More specifically, what is the flux per wavelength unit F_λ at $\lambda = 3500$ Å(ultraviolet) from (a) sunlight, and (b) a 100 watt lightbulb at a distance of 1 m with a color temperature of 2800 K.

23. Two stars, both with blackbody spectra, have the same I-band (infrared) absolute magnitude. One has a color temperature of 5000 K, the other 10000 K. Which star has the larger radius? By what factor?

V. Orbits

24. If a satellite has a nearly circular orbit at a certain distance from the Earth's center, it will have a period of revolution equal to one day and thus can appear stationary in the sky above a particular place on Earth. Calculate the radius of the orbit of such a synchronous satellite. (Abell)

25. The minumum energy spacecraft trajectory from one planet to another is an elliptical orbit that lies between the orbits of the two planets, touching the inner planetary orbit at the perihellion of the spacecraft orbit and the outer planetary orbit at aphelion. For such a trajectory from Earth to Jupiter, calculate the perihelion, aphelion, eccentricity, semimajor axis, and period of the spacecraft orbit. How long does it take the spacecraft to get from Earth to Jupiter? (Abell)

26. Show why the times at which a space vehicle can be sent to a planet on a least-energy orbit occur at intervals of a synodic period of the planet. (Recall that the synodic period was defined in Problem Set 1 of the course.) (Abell)

27. Show that the semimajor axis a of an elliptical orbit depends only on the energy E (that is, not on L), and write a formula for a in terms of E, G, μ, and M.

28. Find a formula for the period τ of an elliptical orbit in terms only of E, G, μ, and M.

VI. Tides

29. Find the separation d between two small bodies, each of mass m, lined up with a large body of mass M, at distance R from the nearest of the small bodies, such that the gravitational attraction between the small bodies is just equal to the differential gravitational force between them caused by their attraction to the large body. The answer should be in terms of G, M, m, and R. (Abell)

30. Compute the relative tide-raising effectiveness of the Sun and the Moon. For this approximate calculation, assume that the Earth is 80 times as

massive as the Moon, and the Sun is 300,000 times as massive as the Earth, and that the Sun is 400 times as distant as the Moon.

Answer: Moon is 8/3 times as effective. (Abell)

VII. Binary Systems

31. How far is the barycenter (Center of Mass) from a star of three times the mass of the Sun in a double-star system in which the other star has a mass equal to the Sun's and a distance of 4 AU from the first star? (Abell)

32. The true relative orbit of Xi Ursae Majoris has a semimajor axis of 2.5 arcsec, and the parallax of the system is 0.127 arcsec. The period is 60 years. What is the sum of the masses of the two stars in units of the solar mass?

    Answer: 2.1 solar masses. (Abell)

33. In a particular visual-spectroscopic binary, the maximum value of the radial velocity of one star with respect to the other is 60 km/s, the inclination of the orbital plane to the plane of the sky is 30°, and the period is 22 days. If the stars in the system have circular orbits, what is the sum of their masses?

    Answer: 3.8 solar masses. (Abell)

34. The observed component of a hypothetical astrometric binary system (i.e., where the motion of one star can actually be measured in the sky) is found to move in an elliptical orbit of semimajor axis 1 AU about the barycenter of the system in a period of 30 years. If the visible star is assumed to have a mass equal to that of the Sun, what is the mass of its unseen companion?

    Answer: About 0.11 solar mass. (Abell)

VIII. Stellar Structure

35. Recall that in fusing Hydrogen into Helium

$$4 \times M_{\mathrm{H}} = 4 \times 1.008\mathrm{AMU} \rightarrow 1 \times M_{\mathrm{He}} = 4.0027 AMU \; .$$

How many kilowatt hours could be generated by a fusion reactor that converts 1 kg of H into He? Estimate roughly for how long this could supply power for a city the size of Boston (1×10^6 people).

36. What is the gravitational potential energy of a uniform-density sphere of mass M and radius R? (Hint: You may want to find the analagous problem in your notes or text from an $E\&M$ course.)

37. A block of Carbon is confined to a volume that just contains it at room temperature, and then heated to very high temperature. At what approximate temperature are its gas pressure

$$\left(\frac{\rho}{\mu}\right) kT$$

and radiation pressure

$$\frac{1}{3} a T^4$$

equal?

38. Verify the first 3 terms in the power series expansion for the solution of the Lane-Emden equation,

$$\phi = 1 - \frac{1}{6}\xi^2 + \frac{n}{120}\xi^4 - \cdots$$

39. Show, for a polytrope, that the mean density of the star is

$$\overline{\rho} = \rho_c \left(\frac{3}{\xi_1} \frac{d\phi}{d\xi}\bigg|_{\xi=\xi_1} \right)$$