Astronomy 45. Introduction to Astrophysics

Problem set 4 Due April 12

1. Properties of Orbits

a) Consider the family of elliptic orbits with constant energy, and show that the circular orbit has the most angular momentum.

b) Consider the family of elliptic orbits with constant angular momentum and show that the circular orbit has the most binding energy.

c) Spacecraft are typically able to make short concentrated momentum changes by turning on their thruster (rocket). They are not usually able to thrust continuously at a low level. Describe the sequence of events used by a spacecraft to change from one circular orbit to a higher circular orbit (say at twice the semi-major axis). Be sure to describe when a thrust occurs, and in what direction.

d) A spacecraft of mass M is in a highly elliptical Earth orbit, and it has enough fuel for a single momentum change Δp. Its goal is to break loose from Earth and coast off to infinity using the smallest possible Δp. Is it better for it to make its Δp thrust when it is at perigee (close to Earth) or apogee (far from Earth)? Explain why. Hint: think about *energy.*

2. Energy Lost During Accretion

Suppose that two stars are bound in a binary system, and that one of these stars (the *primary*) is losing mass to the other (the *secondary*) at a rate $\dot{m}$. Take the radius of the secondary star to be R_2, and its mass to be M_2.

a) Show that the rate of heating of the secondary because of conversion of the kinetic energy of the infalling matter to heat is

$$L = \alpha \frac{GM_2 \dot{m}}{R_2}$$

where $\alpha = 1$ if the infalling matter is assumed to be falling in from rest at infinite distance. In a steady state this energy is lost by radiation. In a real binary star system, what processes occur to make $\alpha < 1$?

b) Use the Stefan-Boltzmann law to show that the temperature of the surface of the secondary star is raised to

$$T_2 = \left(\frac{\alpha G M_2 \dot{m}}{4\pi\rho R_2^3} \right)^{1/4}$$

if the energy of the accreting material is distributed uniformly over the surface of the secondary, the internal energy production of the secondary (by the usual thermonuclear processes) is negligible, and the emerging radiation is of black-body form.

c) The Wien displacement law states that the frequency at which most energy is radiated from a black body at temperature T is

$$\nu = 2.8214 \frac{kT}{h} \ .$$

What luminosity and characteristic frequency would be seen if the secondary star resembles the Sun (mass $M_\odot$ and radius $R_\odot$) and is accreting material at a rate $\dot{m} = 10^{-6} M_\odot$ year^{-1}? Compare the accretion luminosity and the ordinary solar luminosity: is the rate of energy input from accretion significant compared with the internal energy generation rate of such a star?

d) More interesting cases arise when the primary is a white dwarf (of mass $\approx 1 M_\odot$ and radius $\approx 10^4$ km) or a neutron star (of mass $\approx 1.4 M_\odot$ and radius ≈ 10 km). In what wavebands (radio, IR, optical, UV, X-ray, or γ-ray) should observations be made if such stars are to be found, if the accretion rate is $\approx 10^{-6} M_\odot$ year^{-1} again? How luminous (in units of $L_\odot$) would such binary systems be?

(Reading: Shu, pp. 77–80-; 193–195)

3. There is some evidence in the fossil record that the day was once as short as half its current length.

a) How close was the Moon then?

b) How many then-days were there in a then-month?

c) How much larger in amplitude were the tides, then, compared to now?

(If you don't understand this problem, you haven't read notes pp. 72–73.)