Astronomy 45. Introduction to Astrophysics

Problem 1

a) Parsec is a unit of length in astronomy defined so that a star at a distance of 1 parsec has a parallax angle equal to 1 arc second. The parallax baseline in this case is 1 AU.

b) Standard candle is a class of astronomical objects all of which have the same known luminosity. Observations of flux from such object allows us to determine its distance using the inverse square law.

c) Specific intensity I_ν is defined as energy per unit frequency, area, time, and solid angle emitted by a source.

d) Boson is any particle that does not obey Pauli exclusion principle, i.e. bosons like to *clump* together. For example, photons and gravitons are bosons. For those of you who studied or will study quantum mechanics, bosons are the particles with integral spin.

e) Hertzsprung-Russell diagram is a plot of stellar luminosties versus their effective blackbody temperatures or colors.

Problem 2

a) $a = \sqrt{2}R_\odot = \sqrt{2} \times 7 \times 10^{10}$ cm/1.5×10^{13} cm/ AU $= 0.0066$ AU.

b) Kepler's law gives us $P^2 = a^3$, where P is measured in years, and a is measured in AU. Then

$$P = 365\,\mathrm{days/yr} \times (0.0066)^{3/2}\,\mathrm{yr} = 0.196\,\mathrm{days}.$$

c) The key here is to realize that the angular diameter of the Sun is 90°. Then

$$\Omega = \int d\Omega = \int_0^{2\pi} \phi \int_0^{\pi/4} d\theta\, \sin\theta = 2\pi(1 - 1/\sqrt{2}) \simeq 1.84\,\mathrm{sterradian}.$$

d) Energy per unit time absorbed by the planet is $F_\odot \pi R^2$, where R is the planet's radius. Energy per unit time radiated by a planet is $4\pi R^2 \sigma T^4$. The two must be the same at equilibrium, so

$$F_\odot = \frac{L_\odot}{4\pi D^2} = 4\sigma T^4,$$

where D is the planet's distance from the Sun.

Now there are two ways to compute T. You can just substitute the value for $L_\odot$ and D and compute T directly. Another way is to scale from a value for the equilibrium temperature of Earth derived in class. $T_\oplus \approx 300\,\mathrm{K}$, so

$$T \simeq T_\oplus \left(\frac{D_\oplus}{D}\right)^{1/2} = 300\,\mathrm{K} \times (0.0066)^{-1/2} \simeq 3700\,\mathrm{K}.$$

Problem 3

a) We know that

$$V - M_V = 5 \log \left(\frac{D}{10\,\mathrm{pc}}\right).$$

Then $D = 10\,\mathrm{pc}\, 10^{(V-M_V)/5} = 10^5$ pc.

b) Again, by the definition of absolution magnitude

$$M_V - M_{V,\odot} = -2.5 \log \left(\frac{F_V}{F_{V,\odot}}\right) = -2.5 \log \left(\frac{L}{L_\odot}\right).$$

We replaced fluxes by luminosities, since both stars are at the same distance (10 pc). Note that we also assumed that luminosities scale as the V-band fluxes. That is not true in general, but good enough for our purposes.

Then

$$L = L_\odot\, 10^{-(M_V - M_{V,\odot})/2.5} \simeq 230 L_\odot = 9 \times 10^{35}\,\mathrm{ergs/s}.$$

c) The observed flux is just

$$F = \frac{L}{4\pi D^2} = \frac{9 \times 10^{35}\,\mathrm{ergs/s}}{4\pi\,(10^5\,\mathrm{pc} \times 3 \times 10^{18}\,\mathrm{cm/pc})^2} = 8 \times 10^{-13}\,\mathrm{ergs/s/cm^2}.$$

d) The average wavelength of the visible light is $\lambda \sim 5500\mathring{A} = 5.5 \times 10^{-5}$ cm. Assuming that the entire flux is radiated in visible energy band, the number of photons observed is

$$n_\gamma = \frac{F}{h\nu} = \frac{F\lambda}{hc} = \frac{8 \times 10^{-13}\,\mathrm{ergs/s/cm^2}\ 5.5 \times 10^{-5}\,\mathrm{cm}}{6.63 \times 10^{-27}\,\mathrm{ergs\,s}\ 3 \times 10^{10}\,\mathrm{cm/s}} =$$

$$n_\gamma = 0.22\ \mathrm{photons/s/cm^2}.$$

Problem 4

a) The total incident flux is equal to the total outgoing flux, so

$$I = x + y.$$

The same requirement has to hold for the Earth's surface, so

$$I + w = z,$$

and for the greenhouse layer,

$$z = x + y + w.$$

Finally, since the greenhouse layer absorbs only 50% and emits equal amounts up and down, we must have $z = 2x$ and $y = w$.

Now we just have to solve this system of equations. It is not hard to see that

$$y = w = I/3; \quad x = 2I/3; \quad z = 4I/3.$$

b) The Earth must emit $4I/3$ as compared with $2I$, when the greenhouse layer is completely absorbing. Since emission scales as T^4, the new temperature is smaller by a factor of $(2/3)^{1/4}$, so $T = 333\,\mathrm{K}\ (2/3)^{1/4} \simeq 301\,\mathrm{K}$.

b) In the IR frequency range there are some transparent windows, as well as almost completely opaque absoption bands. So the average transparency must be somewhere between 0% and 100%.