Astronomy 45. Introduction to Astrophysics

Problem set 2 Due Friday, February 28

1. Since photons arrive statistically, when you expect to observe N photons on the average, you actually get a value drawn from a statistical distribution with a mean of N and a standard deviation $\sqrt{N}$.

a) How many photons do you need to collect to measure the magnitude of a star to an accuracy of ± 0.02 mag?

b) How long an exposure do you need, with a 1 m telescope, to measure the B magnitude of an apparent magnitude $m_B = 20$ star to ± 0.02 mag. (Hint: use the table in §2.2.1 in the notes.)

For even fainter objects, the brightness of the night sky becomes a limiting factor. The darkest (ground-based) night sky has an equivalent brightness in B of one $m_B = 22.5$ star per square arcsecond.

c) What is the night sky brightness I_λ in $\mathrm{erg/(s\,cm^2\,ster\,Å)}$?

d) How many photons do you need from an $m_B = 24$ star to measure its magnitude to ± 0.02 mag? Assume that the starlight is spread out over exactly 1 square arcsecond (this the the best resolution that can be achieved from the ground). (Hint: photons from the star are competing with statistical fluctuations *from the night sky*, not just with their own fluctuations.)

e) What is the exposure time on a telescope of aperture 4 m required for the measurement in part (d)?

2. A 1.2×10^6 g spacecraft is launched from Earth and is to be accelerated radially away from the Sun using a circular solar sail. Assume a flat sail made from a very lightweight material (say mylar) 10 μm thick, with the density $2\,\mathrm{g/cm^3}$.

a) Compute the initial acceleration of the sail *alone* if

1. it is black, so it absorbs the Sun's light;
2. it is shiny, so it reflects the Sun's light.

Does your answer depend on the radius of the sail? Why?

b) Now let's assume that we want the initial acceleration of the sail and the spacecraft *together* to be half the value you found in part (a). What is the required radius of the sail?

You can assume that the Sun is a blackbody of temperature $T_\odot = 5700\,\mathrm{K}$ and radius $7 \times 10^{10}\,\mathrm{cm}$.

3. Telescopes.

a) The atmosphere limits the angular resolution of ground based optical telescopes to $\sim 1''$. What is the smallest diameter a telescope can have for its angular resolution to be limited by the atmosphere and not by the size of the mirror? Why do you think astronomers keep building larger and larger telescopes on the ground (Keck telescope in Hawaii has $D = 10\,\mathrm{m}$)?

b) A CCD detector is mounted at the focus of an f/1.4 reflecting telescope with at 5 m-diameter mirror. The CCD chip contains 800×800 pixels, with each pixel being $15\,\mu\mathrm{m}$ square. What is the angular size of the sky which is imaged on each pixel? What is the total field of view?

c) The Hubble Space Telescope has the effective diameter of 2.4 m and the CCD chip as described in part (b). What must be its effective f-number to achieve diffraction limited resolution at $\lambda = 5000\,\text{Å}$?

4. The Universe is filled with blackbody radiation of temperature 2.7 K. It is called Cosmic Microwave Background and has originated when our Universe was much denser and hotter.

a) How many microwave background photons are there per cubic centimeter in the Universe today?

b) When our Universe was a factor of Λ smaller in linear dimension, these same photons where in a volume Λ^3 smaller, and it turns out that their *temperature* was also larger by a factor of Λ. Compute the radiation density of in the Universe now ($\Lambda = 0$), and in the past when $\Lambda = 1000$.

c) Assume that the mean density of matter in the Universe today is $2 \times 10^{-31}\,\mathrm{g\,cm^{-3}}$. According to special relativity, each particle has a *rest energy* associated with it which is equal to $E = mc^2$, where m is the mass of the particle. Using this fact, compute at what redshift factor Λ the photon energy density was greater than the energy density in matter.