Astronomy 45. Introduction to Astrophysics

Problem set 3 Due Monday, March 17

1. Elliptical Orbits

Show that the total energy E and angular momentum L for a bound two-body system in an elliptical orbit with an eccentricity e and semimajor axis a are given by the following expressions:

$$E = -\frac{GM\mu}{2a} \quad \text{and} \quad L = \mu\sqrt{GMa(1-e^2)} = \mu\sqrt{GMr_0},$$

where M is the sum of the masses of the two bodies, and μ is the reduced mass. Thus, E depends only on the length a, and L depends only on the length r_0, which has an equally simple geometrical interpretation as the semi-latus rectum. Hint: use the fact that both of these quantities are conserved at any point in the orbit.

2. Properties of Orbits

a) Consider the family of elliptical orbits with constant energy, and show that the circular orbit has the most angular momentum.

b) Consider the family of elliptical orbits with constant angular momentum and show that the circular orbit has the most binding energy.

c) Spacecraft are typically able to make short concentrated momentum changes by turning on their thruster (rocket). They are not usually able to thrust continuously at a low level. Describe the sequence of events used by a spacecraft to change from one circular orbit to a higher circular orbit (say at twice the semi-major axis). Be sure to describe when a thrust occurs, and in what direction.

d) A spacecraft of mass M is in a highly elliptical Earth orbit, and it has enough fuel for a single momentum change Δp. Its goal is to break loose from Earth and coast off to infinity using the smallest possible Δp. Is it better for it to make its Δp thrust when it is at perigee (close to Earth) or apogee (far from Earth)? Explain why? Hint: think about *energy*.

d) Communications and weather satellites are often placed in *geosynchronous* "parking" orbits above Earth. These are orbits where satellites can remain fixed above a specific point on the surface of Earth. At what altitude must these satellites be located? Is it possible for a satellite in a geosynchronous orbit to remain parked over *any* location on the surface of Earth? Why or why not?

3. Sun-Jupiter System

Jupiter is the largest planet in the solar system with mass $M_J = 0.95 \times 10^{-3} M_\odot$ and radius $R_J = 7.14 \times 10^9\,\mathrm{cm}$. The semimajor axis of Jupiter's orbit is $a = 5.2\,\mathrm{AU}$, its orbital eccentricity is $e = 0.048$, and its orbital period is $P = 11.86\,\mathrm{yr}$.

a) Calculate the total angular momentum of the Sun-Jupiter system, using the result from Problem 1.

b) Estimate the contribution the Sun makes to the total orbital angular momentum of the Sun-Jupiter system. For simplicity assume that the Sun's orbital eccentricity is $e = 0$. Hint: First find the distance of the center of the Sun from the center of mass.

c) Estimate the contribution that the orbit of Jupiter makes to the total angular momentum, also assuming that $e = 0$.

d) Recall that the moment of inertia of the solid uniform sphere of mass m and radius r is given by $I = \frac{2}{5}mr^3$. Assuming (incorrectly) that both the Sun and Jupiter rotate as solid uniform spheres, calculate the approximate values for the rotational angular momentum of the Sun and Jupiter. Take the rotation periods of the Sun and Jupiter to be 26 days and 10 hours respectively.

e) What part of the Sun-Jupiter system makes the largest contribution to the *total* angular momentum (total orbital angular momentum + total rotational angular momentum)?