Astronomy 45. Introduction to Astrophysics

Problem set 5 Due Friday, April 18

1. Polytropes

a) Solve the Lane-Emden equation analytically for a special case $n = 0$. Make sure you impose proper boundary conditions at $\xi = 0$. As a check, compute ξ_1 and ϕ_1' and compare them to the values given in the table on page 131.

b) Compute the expressions for $\rho(r)$, $P(r)$, and $T(r)$ in terms of the total mass M and radius R of the star. Based on your result, how would you describe an $n = 0$ polytrope model?

2. White Dwarfs
Suppose a sequence of completely degenerate white dwarf stars has been constructed for different central densities for a composition of pure ${}^{12}_{6}$C. Consider a new sequence of models composed of pure ${}^{56}_{26}$Fe.

a) How are the physical parameters $P(r)$, $\rho(r)$, $m(r)$, and R in the two sequences related? (Compare the models corresponding to the same central density.) You can give either qualitative or quantitative answer.

b) How would the Chandrasekhar mass change from one case to another? For extra credit, try to compute the ratio $M_{Ch}(\mathrm{C})/M_{Ch}(\mathrm{Fe})$.

Incidentally, no white dwarfs composed of iron exist in nature.

3. Supernovae and Inverse β decay
A type II supernova can be viewed as a degenerate white dwarf core collapsing to form a neutron star through inverse β decay. The rest of the star forms an envelope around the core which is ejected after the collapse.

a) For a stellar core of mass $\sim 1.4 M_\odot$ consisting initially of protons and electrons calculate the total energy carried away by neutrinos created in the reaction

$$p^+ + e^- \rightarrow n + \nu_e,$$

if each neutrino has an energy ~ 1 MeV.

b) Compute the initial and final radii of the core (use a non-relativistic equation of state both for a white dwarf and a neutron star) and calculate the gravitational energy lost in the collapse. How does it compare to the energy lost through neutrinos?

c) Assume that the energy not carried away by neutrinos is used in ejecting the stellar envelope. Estimate the resulting velocity of the envelope material assuming that the initial mass of the star is $10M_{\odot}$. Is it a significant fraction of the speed of light?

d) The expanding envelope inevitably collides with the surrounding interstellar medium. Since the expansion velocity is so large, a shock front forms, that essentially transforms kinetic energy of the expansion into thermal energy. Using your result from part (c), estimate the temperature of the material behind the shock front. Will electrons and protons have the same temperature? What frequency band should we use to observe such shock front? (Keep in mind that it is primarily electrons that emit radiation.)